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Historical Growth of Quantities Affecting On-Orbit Collision Hazard

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Over the last 34 years over 22,000 man-made Earth-orbiting objects have been cataloged by the U.S. Space Command Space Surveillance Center. Nearly 7,000 of these objects remain in orbit while only 6% are operational satellites. The thousands of pieces of derelict hardware and fragmentation debris pose a collision threat to each other and the several hundred functioning payloads. The probability of collision (PC) between an operational satellite in low-Earth orbit and the orbital debris environment is mainly a function of spatial density of the debris population, collision cross section, relative impact velocity, and mission duration. Routinely, only changes in spatial density are used to account for changes in the PC as a function of altitude and time. Historical changes to the on-orbit population that would affect the relative velocity and collision cross section are identified. Changes in the relative kinetic energy of an impact are also calculated. The analysis shows that the average relative velocity, collision cross section, and relative energy of impact have increased over time due to changes in the cataloged population. Whereas changes in spatial density are found to have a dominating effect on PC values, the other factors studied have a small effect on the PC but substantially influence the energy of representative impacts. This paper examines the relative importance of factors that influence the collision hazard from the resident on-orbit population.

Nomenclature

AC = collision cross-section, km^2
CS = physical characteristic size, km^2
M = mass, kg
MI = mutual inclination
M_p = mass of impactor
$P_{n>0}$ = probability of one or more collisions
SPD = spatial density, objects/km^3
T = time at risk, s
VO = orbital velocity, km/s
VR = relative velocity, km/s
XC = physical target cross section, km^2

Introduction

THE probability of collision (PC) between two objects in low-Earth orbit (LEO) is calculated using an equation based on the kinetic theory of gases and the Poisson probability of one or more collisions[1]:

$$P_{n>0} = 1 - P_0 = 1 - \exp(-VR*AC*SPD*T) \qquad (1)$$

For LEO missions, a relative velocity VR of 10 km/s is usually used as an average while the parent (target) satellite's physical cross-section XC is used as the collision cross section AC. As the time in orbit T increases, the probability of a collision also increases. Just as the number of cataloged objects has changed over the years, so have the characteristics of this population. The changes in number drive the spatial density SPD terms; this paper investigates how changes in the on-orbit LEO population characteristics have affected VR and AC.

The LEO region (up to 127 min orbital period) is divided into LEO1 (up to 105 min) and LEO2 (105–127 min). The LEO cataloged population growth has been affected by major breakup events, routine launch activities, and varying solar activity while it has averaged about 190 new objects per year. LEO1, being a subset of LEO, has grown at an average of about 120/year. The LEO2 cataloged population exhibited a significantly different growth pattern over time than either LEO or LEO1.[2] The major reason for the leveling of the population growth in LEO2 was the reduction of significant breakup events in that region. The delimiting of growth rate patterns by specific orbital regimes more accurately describes the state and change in the environmental hazard to a satellite in a given orbit.

The discovery of this difference prompted the present analysis to more fully characterize on-orbit risk from debris as a function of orbital regime and time. Four example satellite missions will be used in this paper to quantify these effects (see Table 1).

The PC for each of these four satellites will be calculated for 1-yr stays in orbit for 1970 and 1989. The historical changes in relative velocity for a satellite/debris encounter and the average (physical) size of the on-orbit population will then be incorporated into the PC calculations. Finally, the relative energy of an impact will also be evaluated where the kinetic energy is given by $0.5 * Mp * VR^2$.

Table 1 Example satellite missions

No.	Altitude, km	Orbital regime	Inclination, deg	Physical cross section, m^2
1	450	LEO1	28	200
2	600	LEO1	90	20
3	1100	LEO2	81	10
4	1400	LEO2	65	20

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Table 2 "Simplified" PC calculations

No.	*SPD* 1970	*SPD* 1989	PC/yr 1970	PC/yr 1989	1970–1989, %
1	4.43×10^{-10}	1.13×10^{-8}	2.79×10^{-5}	8.08×10^{-5}	189
2	2.04×10^{-9}	4.08×10^{-9}	1.29×10^{-5}	2.57×10^{-5}	99
3	1.36×10^{-9}	4.58×10^{-9}	4.29×10^{-6}	1.44×10^{-5}	236
4	6.81×10^{-10}	5.97×10^{-9}	4.30×10^{-6}	3.77×10^{-5}	777

Spatial Density

Figure 1 plots the *SPD* values in LEO for 1970 and 1989. By using an average relative velocity of 10 km/s and the parent satellite's physical cross section (termed the "simplified" analysis), Table 2 may be generated to summarize the PC calculations.

The probability of collision for the four example LEO satellites has increased 99–777% (from double to a factor of nine). The largest increases occurred in LEO2, which is less affected by increased solar activity. Satellite 1 (the space station) has a larger PC than the other three satellites mainly due to its larger size. It should be reiterated that these PC values are only for trackable debris, corresponding to impactors having characteristic dimensions on the order of 10 cm and larger.

Relative Velocity

The relative velocity between two objects in circular orbits is a function of orbital velocity *VO* and mutual inclination *MI* by the Law of Cosines:

$$VR = \{2*VO^2*[1 - \cos(MI)]\}^{1/2} \tag{2}$$

The mutual inclination between two LEO objects may span a large range of values depending on the inclinations and right ascensions of each of the objects individually plus the latitude of the encounter. The maximum *MI* between two direct LEO orbits is the sum of their two inclinations whereas the minimum is their difference. An encounter between a direct and a retrograde object requires the use of complementary angles to calculate the *MI*. Thus, the mutual inclination for random collision encounters is a function of the inclinations of the objects in those orbital regimes. Figure 2 shows the growth of the populations in LEO1 and LEO2 by inclination groupings.

Table 3 portrays the inclination distribution as a function of time and orbital regime. Each inclination family is assigned a typical value characteristic of that grouping of satellites. This typical value is then used with the percentage of cataloged objects to create a distribution of satellites from which a range of relative velocities may be computed for a target satellite. Figure 3 plots the average relative velocity values for a range of target satellite inclinations for LEO1/LEO2 and 1970/1989. The range of inclinations is given in the first column with the bracketed term representing the average inclination for each range. There is an expected trend toward higher relative velocities for higher inclination orbits. In addition, for lower inclinations there is a tendency toward increasing relative velocities from 1970 to 1989 and from LEO2 to LEO1. For low inclination LEO1 satellites, there has been about a 500 m/s increase in relative velocity over the last two decades

Table 3 Inclination distributions: number of objects (% of population)

Incl., [deg]	LEO1-1970	LEO1-1989	LEO2-1970	LEO2-1989
0–30 [20]	12 (1%)	33 (1%)	12 (1%)	92 (4%)
30–60 [45]	84 (7%)	167 (7%)	137 (14%)	93 (4%)
60–75 [68]	670 (58%)	885 (35%)	403 (41%)	982 (41%)
75–90 [80]	105 (9%)	706 (28%)	105 (11%)	362 (15%)
90–105 [98]	268 (23%)	747 (29%)	304 (31%)	833 (35%)
+ 105 [110]	8 (1%)	18 (1%)	11 (1%)	14 (1)
Total	1147	2256	972	2376

Table 4 Relative velocities for example satellites, minimum/average/maximum km/s

No.	1970	1989	Change	Change, %
1	6.8/9.3/11.8	7.6/9.8/12.1	0.8/0.5/0.3	11.8/5.4/2.5
2	5.9/10.4/14.8	6.4/10.6/14.9	0.5/0.2/0.1	8.5/1.9/0.7
3	6.0/10.1/14.1	6.4/10.3/14.2	0.4/0.2/0.1	6.7/2.0/0.7
4	5.2/9.3/13.3	5.7/9.5/13.4	0.5/0.3/0.1	9.6/3.3/0.8

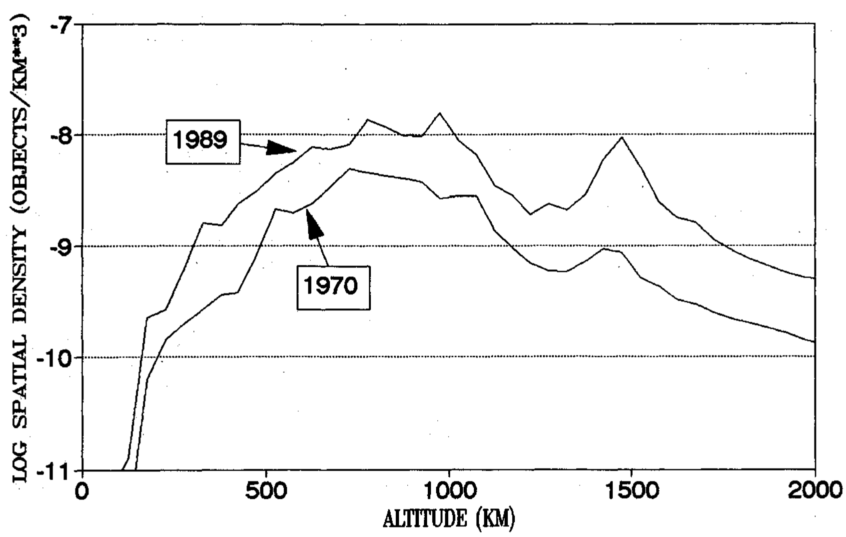
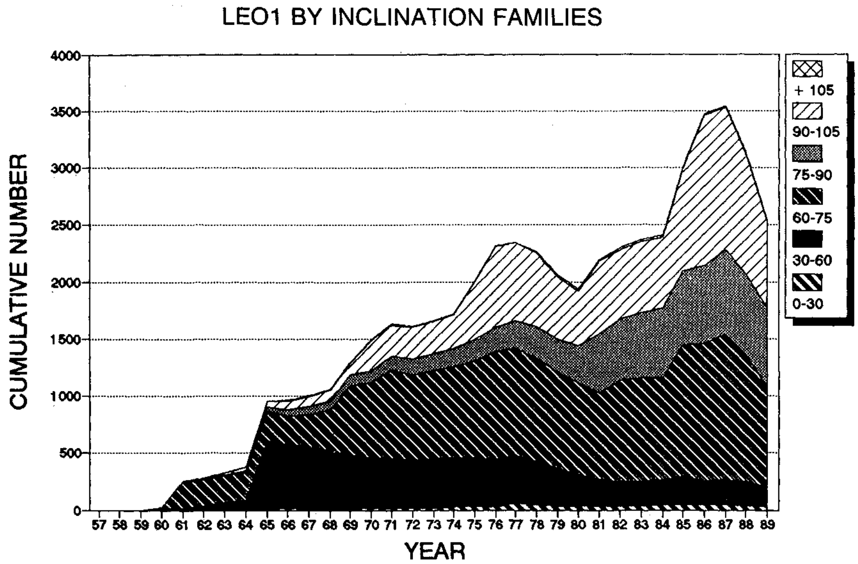


Fig. 1 From 1970 to 1989 the LEO spatial density has increased by a factor of three on the average.



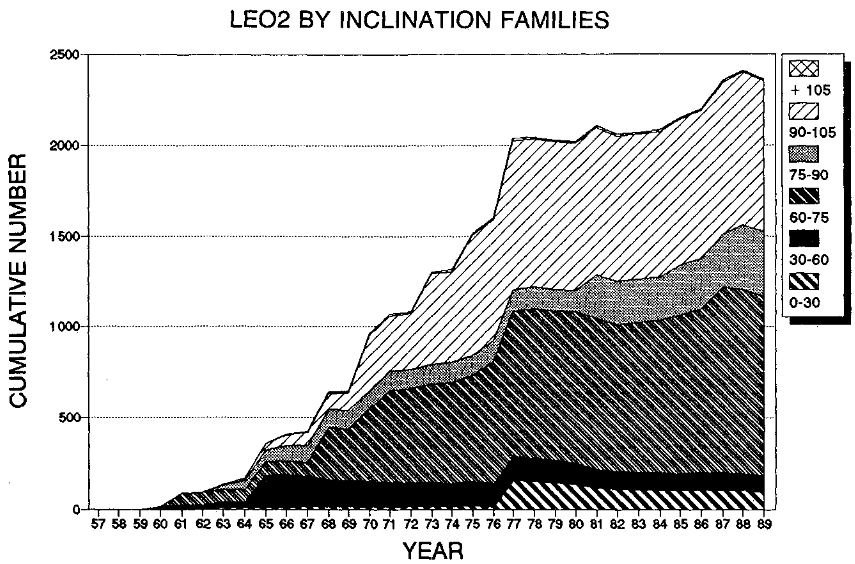


Fig. 2 There is a trend toward more higher inclination orbits in LEO.

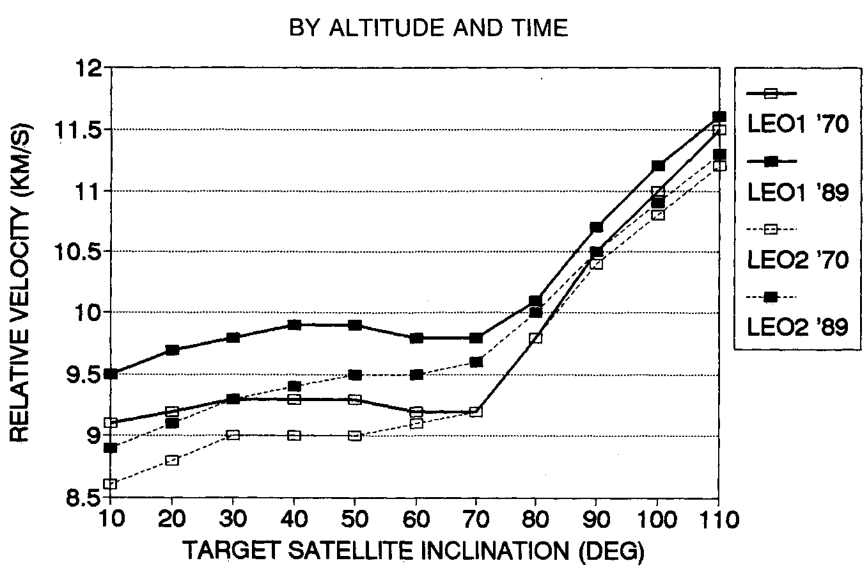


Fig. 3 The average relative velocity increases for higher inclination orbits and has increased more significantly over time at the lower inclinations.

(about a 6% increase). The increase for similar LEO2 objects is only about 300 m/s (about a 3.5% increase).

A more detailed analysis of four example satellites shows similar trends. Table 4 compiles that data.

In all cases, the minimum relative velocity values increased more than the average relative velocities. The overall trend toward higher relative velocity terms is largely the result of more higher inclination (such as polar) satellites and breakups in these orbits.

Cross-Sectional Area

The collision cross section used in Eq. (1) should consider both the physical cross section of the target satellite size XC and the characteristic size CS of the on-orbit population that might impact it. The resulting AC term is calculated as

$$AC = [(XC)^{1/2} + (CS)^{1/2}]^2 \tag{3}$$

The growth of cataloged objects by orbital regime and size (radar cross section) is plotted in Fig. 4. The radar cross section is a function of many parameters and may have a direct correlation to physical cross section and mass. Conversions from radar cross sections (RCS) to mass will be used later to calculate relative kinetic energy values. The present analysis will assume that the physical cross section and RCS are comparable.

Table 5 RCS growth patterns: numbers, %

RCS, m^2	LEO1-1970	LEO1-1989	LEO2-1970	LEO2-1989
0.0-0.1	343 (33%)	710 (28%)	263 (35%)	596 (26%)
0.1-1.0	492 (47%)	843 (34%)	353 (46%)	1027 (44%)
1.0-10	141 (14%)	408 (16%)	110 (1%)	527 (23%)
+ 10	65 (6%)	557 (22%)	37 (5%)	171 (7%)
	1041	2518	763	2321
Average, m^2	3.93	7.26	2.02	2.57

Table 5 tabulates data on the growth of the LEO1 and LEO2 orbital regimes by RCS groupings and average RCS value.

Note that the number of objects tallied in Table 5 differs from Table 3 since many objects with valid element sets may be assigned default RCS values because of incomplete or poor data on the objects. The average RCS in LEO1 increased by 85% from 1970–1989 whereas in LEO2 the change was 27%. (The number increases in LEO1 and LEO2 were 142 and 204%, respectively. These values roughly equate to increases in *SPD* discussed earlier.)

The data in Table 5 must be considered with a number of caveats. To insure that the change in RCS average is "real," 10 objects (rocket bodies, payloads, and debris) were selected

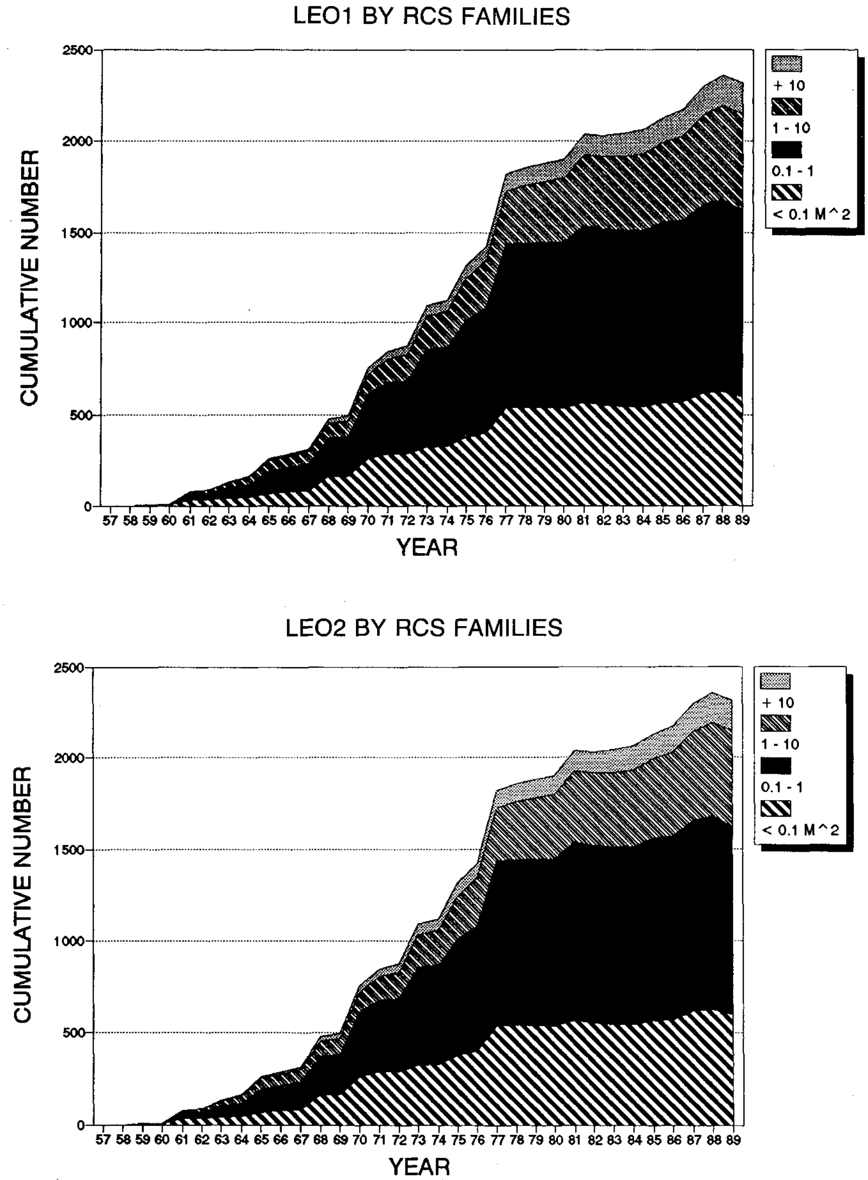


Fig. 4 The size of the average cataloged object has also increased during the last two decades.

Table 6 PC calculations—summary

No.	Simplified, 1970/1989	Complete, 1970/1989	Change, 1970/1989
1	$2.79 \times 10^{-5}/8.08 \times 10^{-5}/189\%$	$2.60 \times 10^{-5}/7.91 \times 10^{-5}/204\%$	$-6\%/-2\%$
2	$1.29 \times 10^{-5}/2.57 \times 10^{-5}/99\%$	$1.36 \times 10^{-5}/2.90 \times 10^{-5}/113\%$	$+5\%/+13\%$
3	$4.29 \times 10^{-6}/1.44 \times 10^{-5}/236\%$	$4.41 \times 10^{-6}/1.60 \times 10^{-5}/263\%$	$+3\%/+11\%$
4	$4.30 \times 10^{-6}/3.77 \times 10^{-5}/777\%$	$4.01 \times 10^{-6}/3.60 \times 10^{-5}/808\%$	$-7\%/-5\%$

Table 7 Relative kinetic energy at impact, kJ

Satellite	50 g, 1970/1989	Change, %
1	2.16/2.40	11.1
2	2.71/2.81	3.7
3	2.55/2.65	3.9
4	2.12/2.26	6.6

in LEO1 and LEO2. RCS values for these objects in 1975 and 1989 were compared. (RCS values were not available for before 1975.) It was found that the RCS for the objects over this time period increased by 100% (doubled) in LEO1 and by over 200% (tripled) in LEO2, yielding an average increase of 160% (more than doubled). This analysis also showed that RCS values of larger objects (>10 m^2) increased by only 25% whereas RCS values of smaller objects more than doubled.[3] This limited survey casts doubts on the significance of conclusions based on the data in Table 5. This exercise highlights a major difficulty in orbital debris analysis—the limited usefulness of RCS data.

Total PC Calculations

The PC per year for each of the four example satellites may be computed using the "simplified" approach (from Table 2) and accounting for changes in VR and AC due to historical trends of the on-orbit population ("complete") (see Table 6).

Using the complete analysis, the PC increase from 1970 to 1989 was very similar in magnitude to that in the simplified analysis. The PC values using actual values for AC and VR were generally within 10% of the simplified analysis. The relative velocity terms dominated the differences in the correlation: the satellites with a VR below 10 km/s had lower complete PC values and vice versa.

Energy of Impact

The PC value provides one measure of hazard, but the relative kinetic energy of the impacting object weighted by the probability of collision results in a measure of average kinetic energy delivered to a satellite's structure by the trackable environment. Table 7 lists the relative kinetic energy for an impact by a 50-g fragment to each of the four satellites considered in this study. The results show that there is only a small increase due to changes in the inclination distribution.

Conclusions

The preceding analyses, while sometimes revealing startling growth in PC or certain other quantities, exhibit the relative importance of the physical traits of object size, relative velocity, and spatial density. In the case of the first two variables, the economics of space transportation and the physics governing orbital mechanics play the dominant role. For example, for trackable objects, there will be no significant gain in average interaction area until the advent of the international space station in the latter part of this decade (since major spacefaring powers appear to have reached a static point with the classes of spacecraft launched into LEO on a regular basis, the Soviets possessing the largest with the Cosmos 1870/Salyut/Mir chassis) or until collisions and/or explosions have produced many more large fragments in LEO. This seemingly obvious phenomenon may be counteracted by a rapid proliferation of lightsats, as many sources are predicting. Also the entire range of possible relative velocities $(0, 2VO)$ is currently available, at least to those objects in high inclination ($i>60$ deg) orbits; particulars depend only on the collision pair's orbital inclinations, their right ascensions, and the latitude of the prospective event. This may be seen by examining the higher-order derivatives of Eq. (2) with respect to MI. Again, only changes in space utilization practices, e.g., launch inclination, will tend to alter the actual per object-pair distribution of VR. One is therefore left with the accumulation of space objects as presenting the major threat of collision, due to the dependence of Eq. (1) on SPD.

The change in the spatial density for a given satellite altitude is the primary factor for the PC for a given target satellite. Thus, the accuracy of probability of collision calculations are mostly a function of spatial density data. The variations calculated in RCS values over time are less than the uncertainty in these measurements so no general trend can be identified. This aspect of the study requires a more in-depth look. Variations in the inclination distribution has a measurable but secondary effect on PC values. On the other hand, changes in these parameters of the on-orbit population may play a significant role in the severity of impacts with man-made debris that a satellite is likely to encounter.

References

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[2]McKnight, D. S., and Johnson, N. L., "Breakups and Their Effects on the Catalog Population," AIAA/NASA/DOD Orbital Debris Conference, April 16–19, 1990.

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